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Some clarification on unit tests:

- Unit test is a practical way to increase our confidence that future changes don't break the intended functionality of existing code.
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- For grading purposes, we will only check the correctness of your test case; however, we encourage you to avoid trivial test cases.
- **General rule of thumb**: if something doesn't exactly match the CHAT manual description in that specific section, leave that part as is in the input; otherwise, modify it as required.

CS 784: Computational Linguistics
Lecture 7: Text Classification

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January 28, 2025

Taxonomy of NLP/CL

Subareas in Linguistics:

- **Morphology**
- Syntax
- Semantics
- Pragmatics

Modeling Approaches:

- Classification
- Language modeling
- Sequence-to-sequence modeling
- Structured prediction

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Tasks: Sentiment Analysis

Sentiment analysis is the task of determining the sentiment of a piece of text.

The sentiment can be positive, negative, or neutral, or it can be more fine-grained.

Text	**Sentiment**
Great service for an affordable price.	Positive
Just booked two nights at this hotel.	Neutral
Horrible services.	Negative

[Source: Socher et al., 2013]

Tasks: Subjectivity vs. Objectivity Judgment

Text	**Label**
The hulk is an anger fueled monster with incredible strength and resistance to damage.	Obj.
In trying to be daring and original, it comes off as only occasionally satirical and never fresh.	Subj.
Solondz may well be the only one laughing at his own joke.	
Obstacles pop up left and right, as the adventure gets wilder and wilder.	

[Source: Pang and Lee, 2004]

Caveat: The task itself is, to some extent, subjective.

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How was the dataset generated?

- IMDB plot summaries: objective.
- Rotten Tomatoes snippets: subjective.

The data could be noisy, but we make this reasonable assumption based on some common wisdom about the platforms.

Tasks: Question Type Classification

Text	Label
Who invented baseball?	Human
CNN is an acronym for what?	Abbreviation
Which South American country is the largest?	Location
How many small businesses are there in the US?	Number
What would you add to the clay mixture to produce bone china?	Entity
What is the root of all evil?	Description

[Source: Li and Roth, 2002]

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Classification results helps QA system to route the question to the appropriate answer extraction module.

Linguistic Acceptability Judgment

A more linguistic oriented task – to determine whether a sentence is acceptable or not (to native speakers).

Text	Label

[Source: Warstadt et al., 2019]

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General tip: Look at your data **a lot** in experiments!

Text Classification: Task Formulation

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At the inference stage,

- Input: text represented as a list of tokens $s = \langle w_1, \ldots, w_n \rangle$.
- Output: a set of categories $Y(s) \subseteq Y$, where Y is the set of all possible categories.

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- Statistical classifier, e.g., naïve Bayes, logistic regression, support vector machines.
- Neural network–based classifier (modern statistical classifiers).

Rule-Based Classifiers

Taking sentiment classification as an example:

- If s contains words in {good, excellent, nice, ...}, then $Y(s) =$ Positive.
- If s contains words in {bad, horrible, awful, ...}, then $Y(s) =$ Negative.

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Check out VADER (Valence Aware Dictionary and sEntiment Reasoner) for a rule-based sentiment analysis toolkit.

Statistical (or Data-Driven) Classifiers

Data-driven modeling of the text classification task.

At the **inference stage**,

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- Input: a set of labeled examples $D = \{(s_1, Y(s_1)), \ldots, (s_n, Y(s_n))\}$.
- Output: a model that can predict the label(s) of text.

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For simplicity, we assume that $Y(s_i)$ is a single category y_i.

We will discuss multi-label classification later.

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$$\text{classify}(s) = \arg\max_{y} score(s, y; \boldsymbol{\Theta})$$

Statistical Classifier: Naïve Bayes

The Bayes' rule:

$$\underbrace{P(y \mid s)}_{\text{classfier}} = \frac{P(s \mid y)P(y)}{P(s)} \propto P(s \mid y)P(y)$$

$s:$ a sentence, $y:$ a category.

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Estimate $P(s \mid y)$ and $P(y)$ for all s and y with training data D.

- $P(y)$: the probability of the category y.

$$P(y) = \frac{\text{count}_D(y)}{\sum_{y'} \text{count}_D(y')}$$

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$$P(s = \langle w_1, \ldots, w_\ell \rangle \mid y) = \prod_{i=1}^{\ell} P(w_i \mid y) \quad P(w \mid y) = \frac{\text{count}_D(w, y)}{\sum_{w'} \text{count}_D(w', y)}$$

Assumption: words in a sentence are conditionally independent given category.

The Bag-of-Words Assumption

The bag-of-words (BoW) assumption: the order of words in a sentence does not matter.

I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun... It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet!

it	6
I	5
the	4
to	3
and	3
seen	2
yet	1
would	1
sweet	1
genre	1
fairy	1
humor	1
have	1
great	1
...	

Laplace Smoothing (Again)

$$P(y) = \frac{\text{count}_D(y)}{\sum_{y'} \text{count}_D(y')} \qquad P(w \mid y) \frac{\text{count}_D(w, y)}{\sum_{w'} \text{count}_D(w', y)}$$

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Solution: Laplace smoothing – add a small constant α to all the counts, and renormalize the probability.

$$P(w \mid y) = \frac{\text{count}_D(w, y) + \alpha}{\sum_{w'} \text{count}_D(w', y) + \alpha|V|}$$

$|V|$: the size of the vocabulary.

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However, if there's a word that never appears in the training data, we may simply ignore it.

Naïve Bayes: Example

Split	Category	Sentence
Training	-	just plain boring
Training	-	entirely predictable and lacks energy
Training	-	no surprises and very few laughs
Training	+	very powerful
Training	+	the most fun film of the summer
Testing	?	predictable with no fun

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Conditional Probabilities (with Laplace smoothing $\alpha = 1$):

$$|V| = 20$$

$$P(\text{predictable} \mid +) = \frac{0+1}{9+20} = \frac{1}{29}$$

$$P(\text{predictable} \mid -) = \frac{1+1}{14+20} = \frac{2}{34}$$

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$$P(+ \mid s) \propto P(+)P(\text{predictable} \mid +)P(\text{no} \mid +)P(\text{fun} \mid +)$$
$$= \frac{2}{5} \times \frac{1}{29} \times \frac{1}{29} \times \frac{2}{29} \approx 3.2 \times 10^{-5}$$

$$P(\text{-} \mid s) \propto P(-)P(\text{predictable} \mid -)P(\text{no} \mid -)P(\text{fun} \mid -)$$
$$= \frac{3}{5} \times \frac{2}{34} \times \frac{2}{34} \times \frac{1}{34} \approx 6.1 \times 10^{-5}$$

Statistical Classifier: Logistic Regression

Base case: binary classification for text classification.

Training data: $D = \{(s_1, y_1), \ldots, (s_n, y_n)\}, y_i \in \{0, 1\} (\forall i)$.

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Each piece of text s_i will be represented as a feature vector $\mathbf{x}_i$.

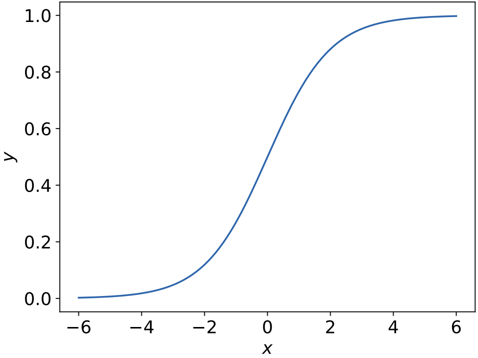
Train a model parameterized by $\mathbf{w}$ to predict the probability of y_i given $\mathbf{x}_i$ with the logistic function:

$$P(y_i = 1 \mid \mathbf{x}_i) = \sigma\left(\mathbf{w}^T\mathbf{x}_i\right) = \frac{1}{1 + \exp(-\mathbf{w}^T\mathbf{x}_i)}$$

$$P(y_i = 0 \mid \mathbf{x}_i) = 1 - \sigma\left(\mathbf{w}^T\mathbf{x}_i\right) = \frac{\exp(-\mathbf{w}^T\mathbf{x}_i)}{1 + \exp(-\mathbf{w}^T\mathbf{x}_i)}$$

$\sigma(\cdot)$: the sigmoid function; $\mathbf{w}, \mathbf{x}_i \in \mathbb{R}^d$.

The Sigmoid function



$$\sigma(x) = \frac{1}{1 + \exp(-x)}$$

It normalizes the output to $[0, 1]$, which has natural interpretation as probability.

Training Logistic Regression

- Training objective: maximize the likelihood of training data
  Assumption: each example is independent and identically distributed (i.i.d.)

$$\begin{aligned}\mathcal{L}(\mathbf{w}) &= \prod_{i=1}^{n} P(y_i \mid \mathbf{x}_i) \\ &= \prod_{i=1}^{n} \sigma(\mathbf{w}^T\mathbf{x}_i)^{y_i}(1-\sigma(\mathbf{w}^T\mathbf{x}_i))^{1-y_i}\end{aligned}$$

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$$\log \mathcal{L}(\mathbf{w}) = \sum_{i=1}^{n} \left[y_i \log \sigma(\mathbf{w}^T\mathbf{x}_i) + (1 - y_i) \log(1 - \sigma(\mathbf{w}^T\mathbf{x}_i)) \right]$$

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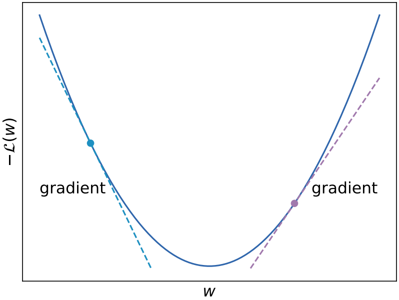
- Maximize the log-likelihood by **gradient ascent**, or minimize the negative log-likelihood loss by **gradient descent**:

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \eta \nabla_{\mathbf{w}} \log \mathcal{L}(\mathbf{w})$$

$$\nabla_{\mathbf{w}} \log \mathcal{L}(\mathbf{w}) = \left[\frac{\partial \log \mathcal{L}(\mathbf{w})}{\partial w_1}, \ldots, \frac{\partial \log \mathcal{L}(\mathbf{w})}{\partial w_d} \right]$$

η: learning rate, t: time step (iteration number).

Gradient Descent: The Idea



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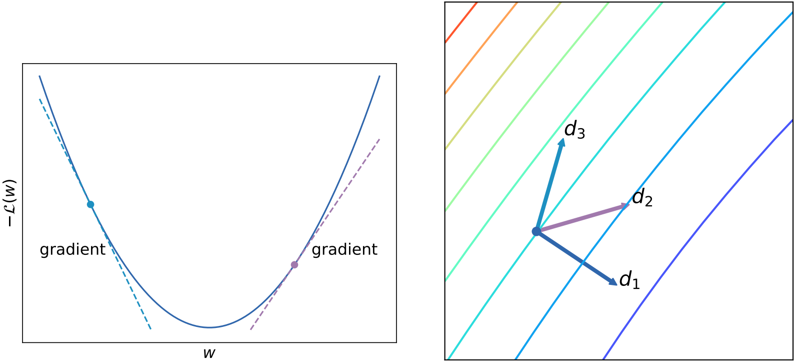

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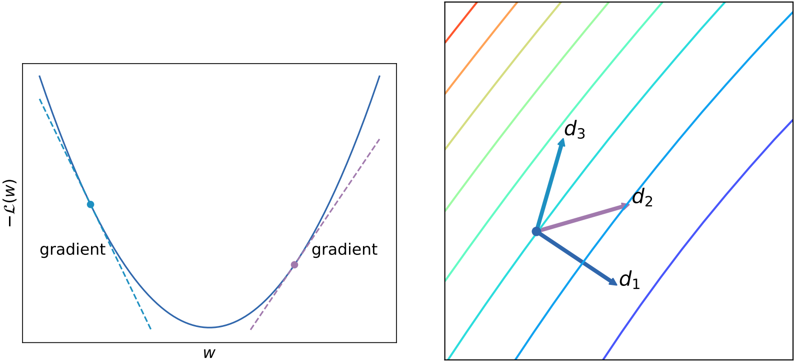

Find the direction of the steepest descent (i.e., the direction that is perpendicular to the contour lines), move slightly along that direction, and repeat until convergence.

See here for a formal proof.

Explain Logistic Regression

$$P(y = 1 \mid \mathbf{x}) = \sigma\left(\mathbf{w}^T\mathbf{x}\right) = \frac{1}{1 + \exp(\underbrace{-\mathbf{w}^T\mathbf{x}}_{\text{linear}})}$$

The weight vector **w** is a set of coefficients that are learned during training.

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Larger absolute values of a w_i means the feature x_i has a larger impact on the prediction.

Generative vs. Discriminative Models

Generative models: model the joint distribution of the input and output $P(x, y)$. Once we have this distribution, we may **generate new data** by sampling from it.

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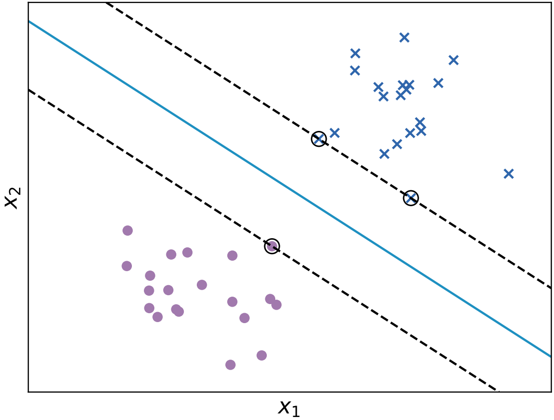
Naïve Bayes vs. logistic regression: which one is generative and which one is discriminative?

Support Vector Machines (SVMs)

Suppose the data is linearly separated, an SVM finds the hyperplane that maximizes the margin between the two classes.



SVM: Formulation

We have the training data $D = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_n, y_n)\}$, where $\mathbf{x}_i \in \mathbb{R}^d$ denotes features and $y_i \in \{-1, 1\}$ denotes a label.

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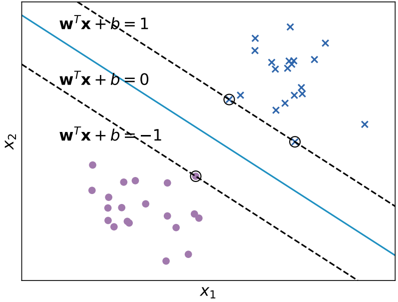
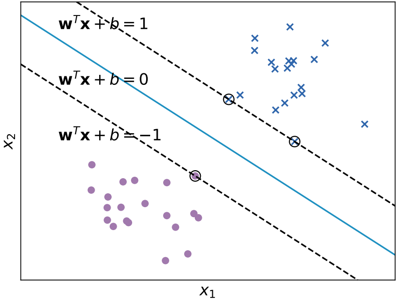
In the inference stage, we predict the label of a new data point $\mathbf{x}$ by $\text{sign}(\mathbf{w}^T\mathbf{x} + b)$.

SVM: The Idea


$\mathbf{w}^T\mathbf{x} + b = 1$
$\mathbf{w}^T\mathbf{x} + b = 0$
$\mathbf{w}^T\mathbf{x} + b = -1$
x_2
x_1

SVM: The Idea



There are infinitely many (equivalent) ways to write down the decision boundary:

$$x_1 + x_2 - 5 = 0 \qquad \mathbf{w} = [1, 1]^T, b = -5$$

$$2x_1 + 2x_2 - 10 = 0 \qquad \mathbf{w} = [2, 2]^T, b = -10$$

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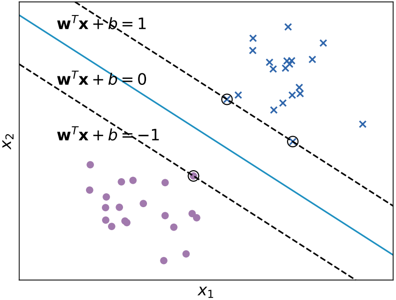
SVM: The Idea



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$$\ldots$$

The $\mathbf{w}$ and b with $\min_i y_i \mathbf{w}^T\mathbf{x}_i + b = 1$ is just one representative within the equivalence class.

SVM: The Representor Theorem

The solution to the problem

$$\mathbf{w}^* = \arg\min_{\mathbf{w},b} \left\{ \frac{1}{2}\|\mathbf{w}\|^2 \right\} \text{ s.t. } y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1, \forall i$$

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Let $\mathbf{w}^* = \mathbf{w}_\mathbf{X} + \mathbf{w}_\perp$, where $\mathbf{w}_\mathbf{X} \in \text{span}(\mathbf{x}_1, \ldots, \mathbf{x}_n)$ and $\mathbf{w}_\perp \cdot \mathbf{w}_\mathbf{X} = 0$ is orthogonal to all $\mathbf{x}_i$.

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Practice: complete the proof.

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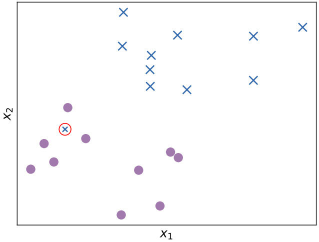

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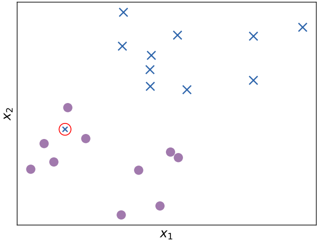

Add **slack variables** ξ_i to allow some misclassification:

$$y_i(\mathbf{w}^T \mathbf{x}_i + b) \geq 1 - \xi_i \quad (\xi_i \geq 0)$$

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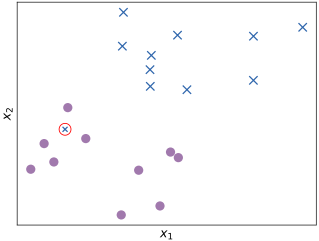

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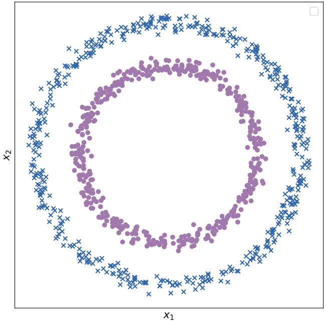
Minimize the SVM loss J with (sub)gradient descent.

$$J(\mathbf{w}, b) = \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{n}\xi_i(\mathbf{w}, b)$$

C: balancing hyperparameter.

More about SVMs

Linear projection is sometimes not a good idea for the original data, but it could be after some (nonlinear) projection.



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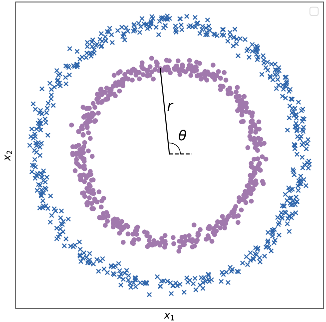

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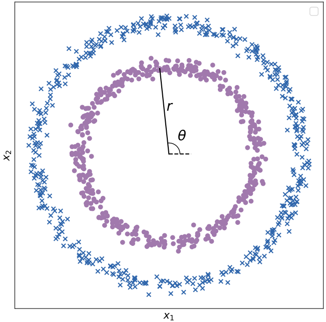

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The **kernel trick**: We don't really need to know the projection function, as long as we can compute the dot product in the projected space.

Readings: PRML Chapter 7.1

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The classifiers we've discussed so far

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For logistic regression and SVM, we've only discussed binary classification.

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Model	**Probabilistic**	**G/D**
Naïve Bayes	Yes	Generative
Logistic Regression	Yes	Discriminative
SVM	No	Discriminative

Generalization to Multiple Classes

There are two common approaches to extend binary classifiers to support K classes:

- **One-vs-Rest**: Train K binary classifiers, one for each class.
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If there is a tie, break it randomly or use any plausible strategy.

Next

Classification with Features from Advanced Neural-Net Structures

$$\text{logits}(\mathbf{x}) = \text{NN}(\mathbf{x}) \in \mathbb{R}^K$$
$$P(y \mid \mathbf{x}) = \text{softmax}(\text{logits}(\mathbf{x}))$$

or

$$P(\mathbf{x}, y) \propto \text{NN}(\mathbf{x}, y)$$