CS 784: Computational Linguistics
Lecture 10: Language Models

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February 24, 2025

Recap: Distributional Semantics

You shall know a word by the company it keeps.

– J. R. Firth, 1957

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- A bottle of **tezgüino** is on the table.
- Everybody likes **tezgüino**.
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- **Tezgüino** is made out of corn.

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This is **language modeling**.

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- Define a statistical model $P_{\boldsymbol{\Theta}}(\mathbf{x})$ ($\mathbf{x}$: string).
- Estimate the parameters $\boldsymbol{\Theta}$ from data (by maximizing likelihood).

$$\boldsymbol{\Theta}^* = \arg\max_{\boldsymbol{\Theta}} \prod_{i=1}^{N} P_{\boldsymbol{\Theta}}(\mathbf{x}_i) = \arg\max_{\boldsymbol{\Theta}} \sum_{i=1}^{N} \log P_{\boldsymbol{\Theta}}(\mathbf{x}_i)$$

Language Modeling as a Foundational Task

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- Machine translation
  $P(\textit{turn the camera off}) > P(\textit{put the camera out})$
- Speech recognition
  $P(\textit{a tomato garden}) > P(\textit{a tornado garden})$
- Grammatical error correction
  $P(\textit{about fifteen minutes}) > P(\textit{about fifteen minuets})$

Language Models: Data Matters

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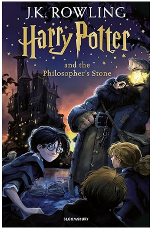

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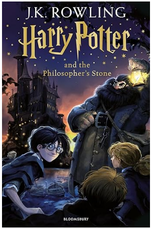

Language models highly depend on their training data that define the population distribution.

Language Modeling: From Word to Sequence Probability

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From a high-level modeling perspective,

- **Autoregressive language models**: compute $P(x_T | x_1, x_2, \ldots, x_{T-1})$.
- **Masked language models**: compute $P(x_k | x_1, \ldots, x_{k-1}, x_{k+1}, \ldots)$ with some tokens masked.

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From a methodological perspective,

- **Count-based** language models: n-gram models.
- **Neural** language models: RNNs, LSTMs, Transformers.

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An **autoregressive language model** computes the conditional probability $P(x_T \mid x_1, x_2, \ldots, x_{T-1})$.

Important detail: remember to model **sequence length** – a special token $\langle \texttt{EOS} \rangle$ is necessary in probabilistic terms!

Modeling Length: The End-of-Sequence Token

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Instead of calculating $P(x_1, x_2, \ldots, x_T)$, we calculate $P(x_1, x_2, \ldots, x_T, \langle \texttt{EOS} \rangle)$ as the sequence probability.

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If there is no $\langle \texttt{EOS} \rangle$ token

$$\begin{aligned} P(x_1, \ldots, x_T) &= P(x_1, \ldots x_{T-1})P(x_T \mid x_1, \ldots, x_{T-1}) \\ &\leq P(x_1, \ldots, x_{T-1}) \end{aligned}$$

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$$P(\textit{The cat is cute.}) \leq P(\textit{The cat is})$$

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$$(\text{length } T = 1) \quad \sum_{\mathbf{x}} P(\mathbf{x}_{1:T}) = \sum_{x_1 \in V} P(x_1) = 1$$

V: vocabulary.

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&(\text{length } T=1) \quad \sum_{\mathbf{x}} P(\mathbf{x}_{1:T}) = \sum_{x_1 \in V} P(x_1) = 1 \\
&(\text{length } T=2) \quad \sum_{\mathbf{x}} P(\mathbf{x}_{1:T}) = \sum_{x_1 \in V} \sum_{x_2 \in V} P(x_1)P(x_2 \mid x_1) \\
&\qquad\qquad\qquad\qquad\qquad\quad = \sum_{x_1 \in V} P(x_1) \left(\sum_{x_2 \in V} P(x_2 \mid x_1) \right) = 1
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Modeling Length: The End-of-Sequence Token

If we have the $\langle \texttt{EOS} \rangle$ token, the sum of sequence probability becomes

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Practice: complete the proof.

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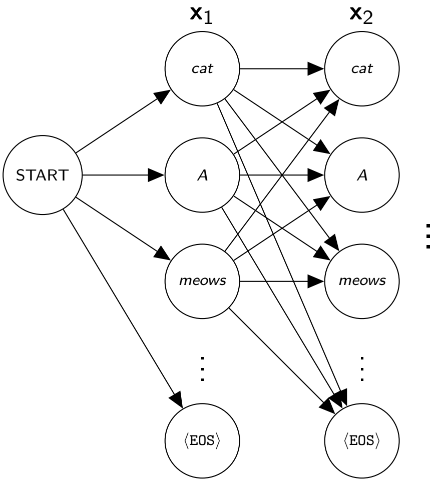

$$
\begin{aligned}
&P(\textit{meows}, \langle \texttt{EOS} \rangle) \\
=&P(\textit{meows})P(\langle \texttt{EOS} \rangle \mid \textit{meows})
\end{aligned}
$$

Each edge represents a (conditional) probability term after factorization.

N-Gram Language Models: The Markov Assumption

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Counting-based methods cannot efficiently model the conditional probability $P(x_T \mid x_1, x_2, \ldots, x_{T-1})$ when n goes large.

$$\begin{aligned} & P(\textit{The cat is cute.}) \\ = & P(\textit{The})P(\textit{cat} \mid \textit{The})P(\textit{is} \mid \textit{The cat}) \\ & P(\textit{cute} \mid \textit{The cat is})P(. \mid \textit{The cat is cute}) \end{aligned}$$

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The **Markov assumption**: the probability of a token only depends on the previous $n-1$ tokens ($n << \text{sequence length } T$).

[Andrey Markov]

N-Gram Language Models: The Markov Assumption

In other words, the **Markov assumption** assumes independence of a token from distant history, conditioning on its close history.

$$P(x_i \mid x_1, x_2, \ldots, x_{i-1}) \approx P(x_i \mid \underbrace{x_{i-n+1}, x_{i-n+2}, \ldots, x_{i-1}}_{\text{always } n-1 \text{ entries}})$$

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We can estimate the conditional probability $P(x_i \mid x_{i-n+1}, x_{i-n+2}, \ldots, x_{i-1})$ by counting the occurrences of n-grams:

$$P(x_i \mid x_{i-n+1}, \ldots, x_{i-1}) = \frac{\text{count}(x_{i-n+1}, \ldots, x_{i-1}, x_i)}{\text{count}(x_{i-n+1}, \ldots, x_{i-1})}$$

Common N-Gram Language Models

- **Unigram language models** (n=1):

$$P(\mathbf{x}) = P(x_1)P(x_2)\ldots P(x_T)$$
$$P(\textit{This is a cute cat}) = P(\textit{This})P(\textit{is})P(\textit{a})P(\textit{cute})P(\textit{cat})$$

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- **N-Gram language models** (n>2): similar to bigram models—should be paired with sparse techniques to store the probabilities.

Sample Sentences from Unigram and Bigram LMs

Both trained on financial news.

Model 1:

fifth an of futures the an incorporated a a the inflation most dollars quarter in is mass thrift did eighty said hard 'm july bullish that or limited the

Model 2:

texaco rose one in this issue is pursuing growth in a boiler house said mr. gurria mexico 's motion control proposal without permission from five hundred fifty five yen outside new car parking lot of the agreement reached this would be a record november

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Generating from a Language Model

Taking bigram LMs as an example,

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- ...
- Repeat until the $\langle \texttt{EOS} \rangle$ token is generated.

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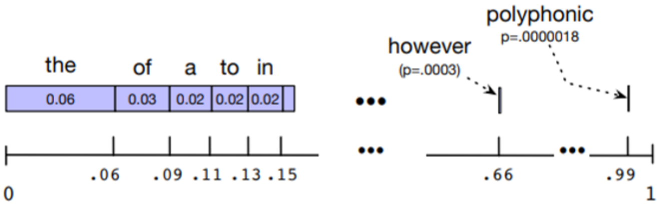
Recap: sampling from a distribution.



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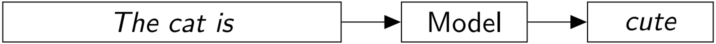
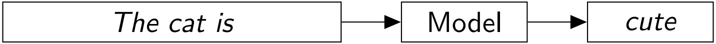


Neural Autoregressive LMs



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Key idea: generate one token at a time.

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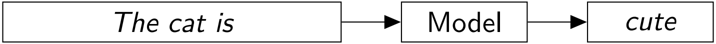

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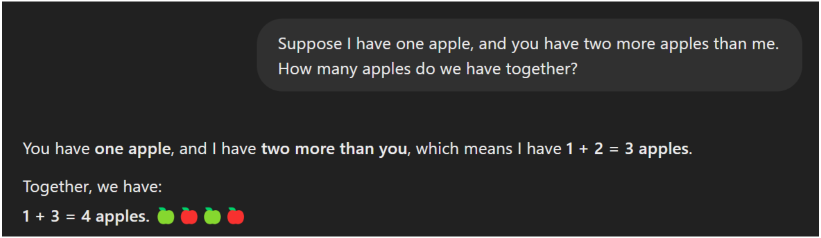
Compared to n-gram LMs, Transformer-based LMs can handle much longer dependencies and generate coherent text.



Neural Autoregressive LMs: Training

Suppose training examples are drawn from an i.i.d. distribution.

Objective: maximize the (log) likelihood of the training data, which can be broken down into token-level probabilities.

$$\begin{aligned}\boldsymbol{\Theta}^* &= \arg\max_{\boldsymbol{\Theta}} \sum_{i=1}^{N} \log P_{\boldsymbol{\Theta}}(\mathbf{x}_i) \\ &= \arg\max_{\boldsymbol{\Theta}} \sum_{i=1}^{N} \sum_{t=1}^{T_i} \log P_{\boldsymbol{\Theta}}(x_{i,t} \mid x_{i,1}, \ldots, x_{i,t-1})\end{aligned}$$

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s: input text, y: output, $\mathbf{\Theta}$: model parameters.

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- From the classification perspective, this is a natural extension of classification.
- From the word embeddings perspective, we are now allowed to use more complex models $score(s, y; \mathbf{\Theta})$.

Generating Text from Language Models

Given a well-trained language model $P_{\Theta}(x_t \mid x_1, \ldots, x_{t-1})$, how do we generate text?

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At each time step, we have several options:

- **Greedy decoding**: choose the token with the highest probability.
- **Beam search**: keep track of the top-k hypotheses.
- **Sampling**: sample from the distribution.
- **Top-k sampling**: sample from the top-k tokens with the highest probability.
- **Nucleus sampling (top-p) sampling**: sample from the smallest set of tokens whose cumulative probability exceeds a threshold p.

Greedy Decoding

At each time step, choose the token with the highest probability.

Repeat until the $\langle \texttt{EOS} \rangle$ token is generated, or it reaches a maximum length.

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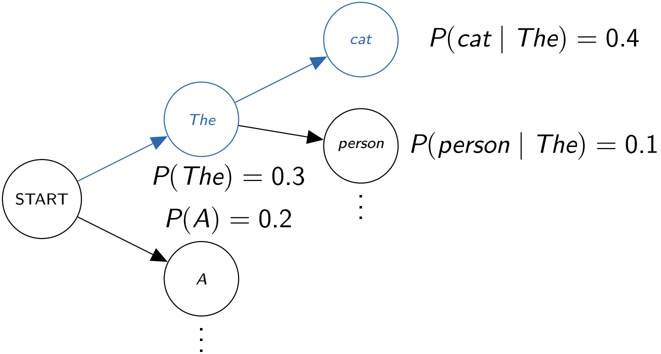

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(Beam size $= 2$)

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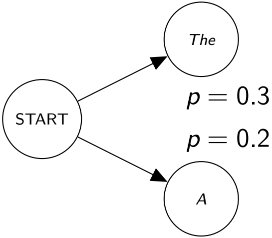

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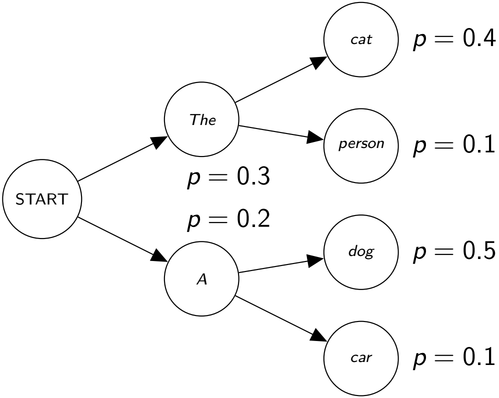

Example
(Beam size = 2)

Step 1:

- The (0.3)
- A (0.2)

Step 2:

- The cat (0.12)
- A dog (0.1)

Greedy Decoding vs. Beam Search

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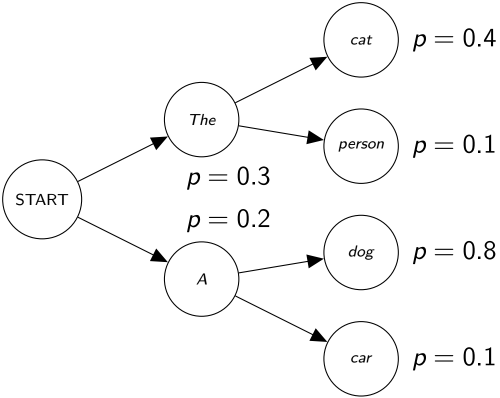

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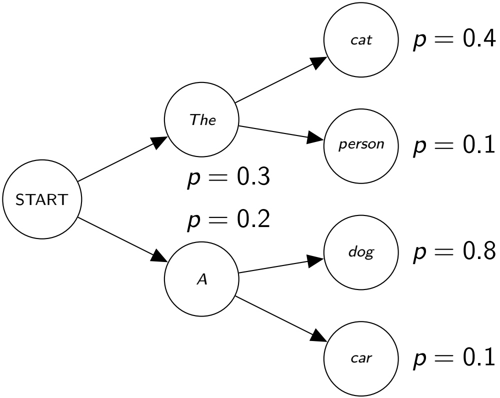
A: Beam search (A *dog*, $P = 0.16$).



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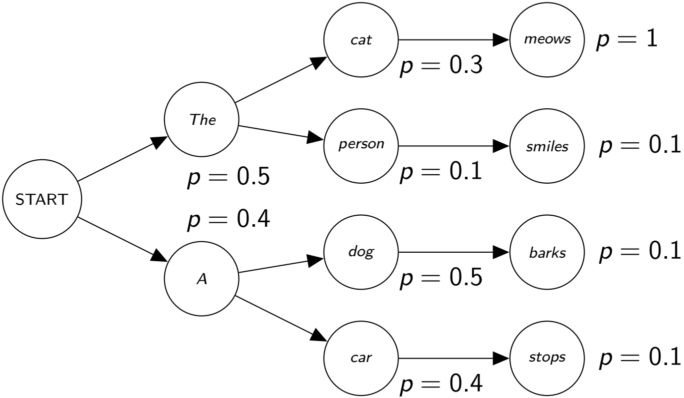

Greedy Decoding vs. Beam Search

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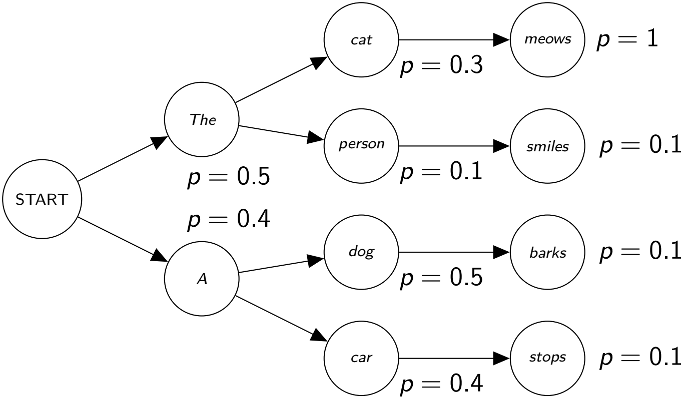
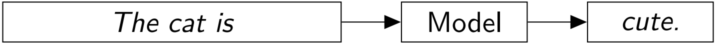


Language Modeling: Summary

Autoregressive language modeling (e.g., GPT, Radford et al., 2018):



Language Modeling: Summary

Autoregressive language modeling (e.g., GPT, Radford et al., 2018):

The cat is → Model → *cute.*

Masked language modeling (BERT, Devlin et al., 2019):

The [MASK] is cute. → Model → *cat*

The [MASK] [MASK] cute. → Model → *cat, is*

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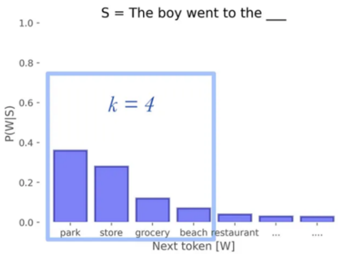

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Top-p (Nucleus) sampling

(Holtzman et al., 2019)

Another way to define the tail of the distribution.

Evaluating Language Models

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Perplexity of Held-Out Data

Log-probability of held-out data $\mathcal{X}$ with model $P_{\mathbf{\Theta}}$:

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Higher the probability of the held-out data means... it's less perplexing to the model.

Next

Masked Language Models, Sequence Labeling