CS 784: Computational Linguistics
Lecture 14: Syntax - Dependency Parsing

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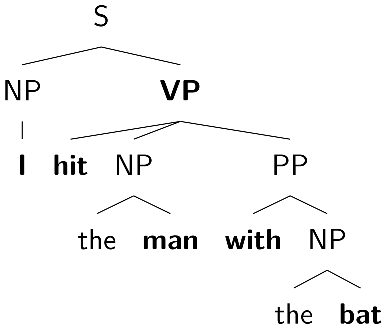
From Constituency to Dependency



Boldfaced words: head of the phrase.

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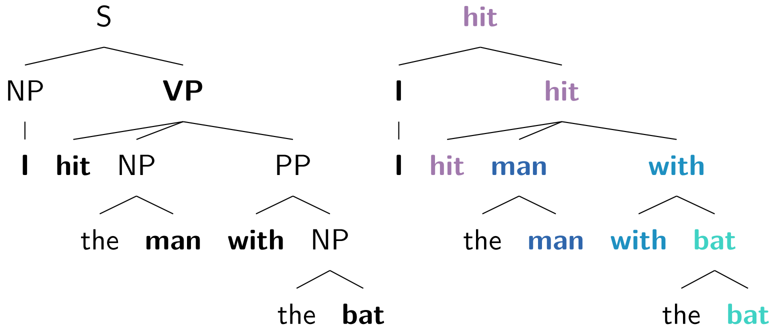

Boldfaced words: head of the phrase.

Propagate the lexical heads up in the tree.

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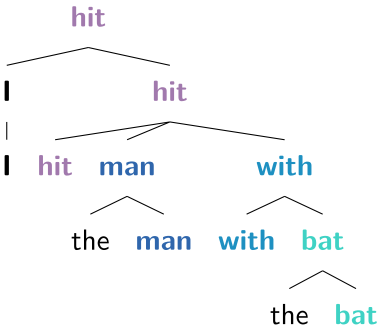

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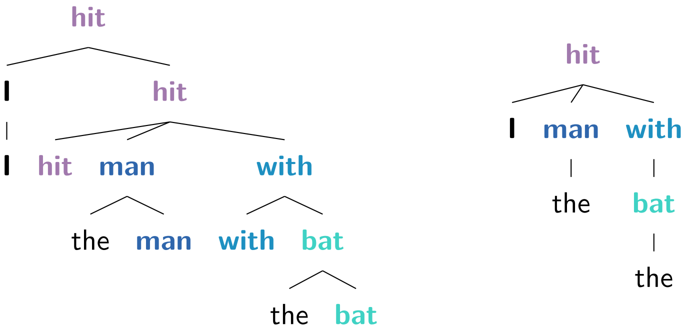
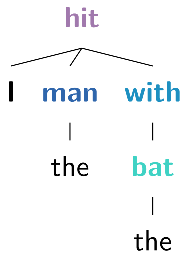


Remove the redundant nodes by keeping the top one.

From Constituency to Dependency



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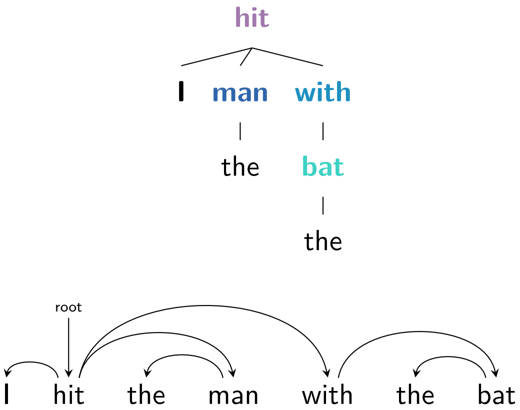
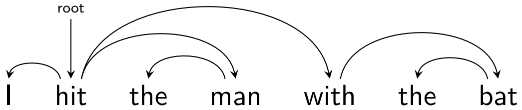


Replace each edge with an arc from the head to the dependent.

Dependency Parse: Properties



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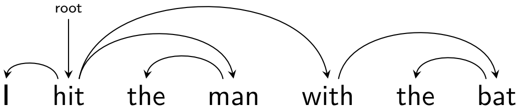

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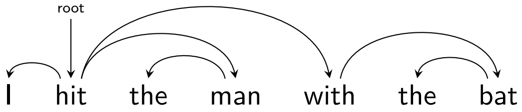

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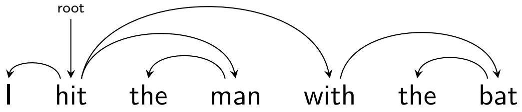

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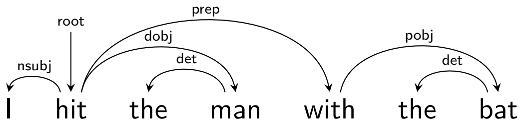

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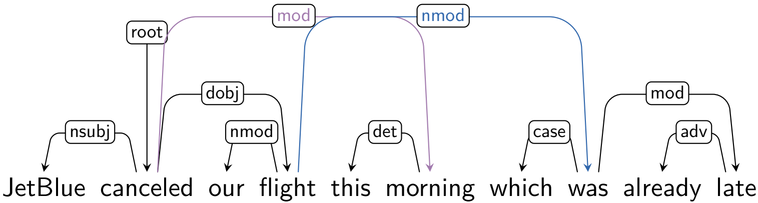
- Each edge can be labeled with a **dependency relation**.

Some Dependency Relations

Causal Argument Relations	Description
`nsubj`	Nominal subject
`dobj`	Direct object
`iobj`	Indirect object
`ccomp`	Clausal complement
`xcomp`	Open clausal complement
Modifier Relations	**Description**
`nmod`	Nominal modifier
`amod`	Adjectival modifier
...	

[Source: SLP3]

Projectivity



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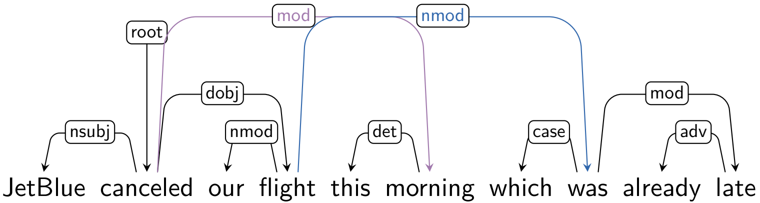

A dependency parse is **nonprojective** if and only if there exist two crossing dependency arcs.

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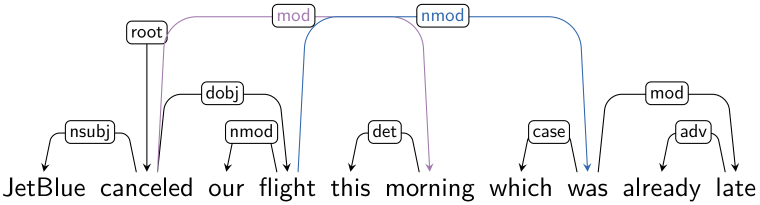

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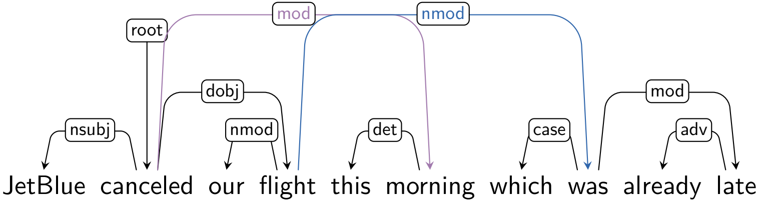

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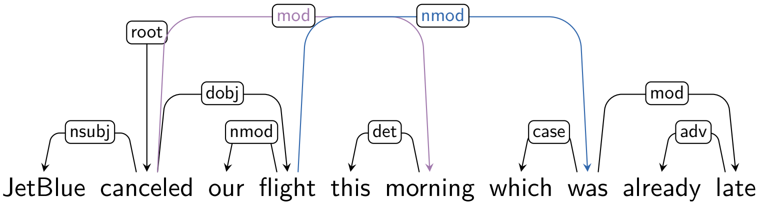

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Languages with relatively free word orders, like Czech, are fairly nonprojective.

Universal Dependencies

The Universal Dependencies (UD) project aims to provide a cross-linguistically consistent treebank annotation scheme.

https://universaldependencies.org/

	Nominals	Clauses	Modifier words	Function Words
Core arguments	nsubj obj iobj	csubj ccomp xcomp		
Non-core dependents	obl vocative expl dislocated	advcl	advmod* discourse	aux cop mark
Nominal dependents	nmod appos nummod	acl	amod	det clf case
Coordination	**Headless**	**Loose**	**Special**	**Other**
conj cc	fixed flat	list parataxis	compound orphan goeswith reparandum	punct root dep

Universal Dependencies: An Intuitive Example

While detailed grammatical realizations differ across languages, the underlying syntactic structure is often similar.

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Shi et al. (2022): multilingual language models enables zero-shot cross-lingual dependency analysis, even for quite different language pairs.

Evaluation of Dependency Parsing

Unlabeled attachment score (UAS): the proportion of words that are assigned the correct head (suppose each word is assigned with one head, and the dummy "root" is considered as a valid head).

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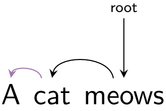
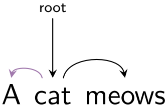


$\mathcal{T}_2$:



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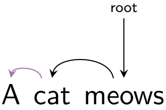

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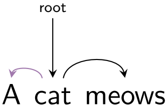

$\text{UAS}(\mathcal{T}_1, \mathcal{T}_2) = \frac{1}{3}$

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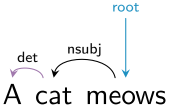
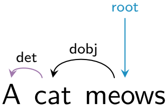
$\mathcal{T}_1$: 

$\mathcal{T}_2$: 

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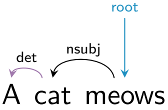

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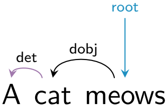

$\text{LAS}(\mathcal{T}_1, \mathcal{T}_2) = \frac{2}{3}$

Recap: The General NLP Problem Formulation

$$\text{parse}(s) = \arg\max_{\mathcal{Y}} \text{score}(s, \mathcal{Y}; \boldsymbol{\Theta})$$

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In practice, we also sometimes only consider **projective** trees.

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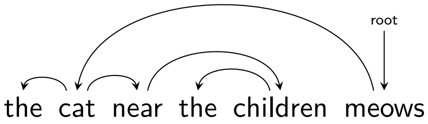
Example: $F[3, 5, 3]$ is the highest scoring tree for the span $[3, 5]$ (near the children) rooted at index 3 (near).



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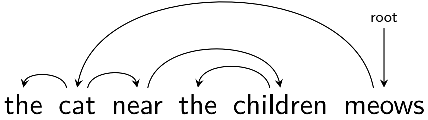

Implicit assumption: when calculating $F[\ell, r, t]$, we are thinking about the hypothetical condition that $[\ell, r]$ is a **constituent**.

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$$F[\ell, r, t] = \max \begin{cases} \max\limits_{\substack{\ell \leq m < t \\ \ell \leq t_\ell \leq m}} F[\ell, m, t_\ell] + F[m+1, r, t] + \text{score}(t \to t_\ell), \\ \max\limits_{\substack{t \leq m < r \\ m \leq t_r \leq r}} F[\ell, m, t] + F[m+1, r, t_r] + \text{score}(t \to t_r) \end{cases}$$

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Key idea: if t is the root of the tree, it must be the root of the left/right subtree as well.

- m: the split point.
- t_ℓ: the head of the left child.
- t_r: the head of the right child.
- $score(t_\ell \rightarrow t_r)$: the score of the arc from t_ℓ to t_r.
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Final Answer: $\max_{1 \le t \le n} F[1, n, t]$.

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Time complexity:

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Time complexity: $\mathcal{O}(n^5)$.

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The Eisner's algorithm (1996) improves the time complexity to $\mathcal{O}(n^3)$ and space complexity to $\mathcal{O}(n^2)$, with some smart realization of the "trapezoid" structure.

Chu-Liu/Edmonds' Algorithm

Proposed independently by Yoeng-Jin Chu and Tseng-Hong Liu (1965) and Jack Edmonds (1967).

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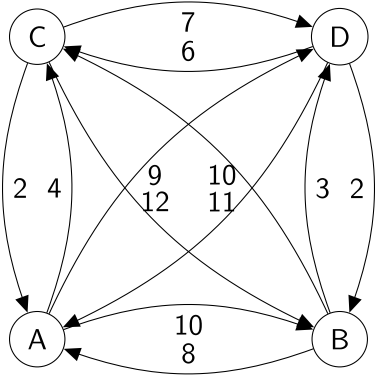
  3. If there's a cycle, contract it into a single node.
  4. Recalculate edge scores in the contracted graph.
  5. Recursively find the best tree in the new graph.
  6. Expand the contracted node back into a cycle.

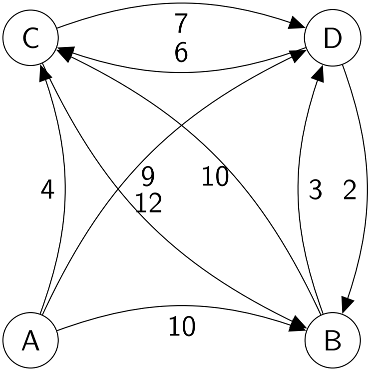
Running Chu-Liu/Edmonds' Algorithm with an Example

Without loss of generality, we assume the root is node A—in practice, we may need to enumerate.



Running Chu-Liu/Edmonds' Algorithm with an Example

Step 1: since we assume A is the root, we remove all incoming edges to A.



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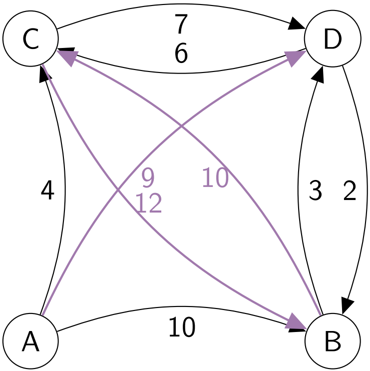
Step 2: find the highest scoring incoming edge for each node.



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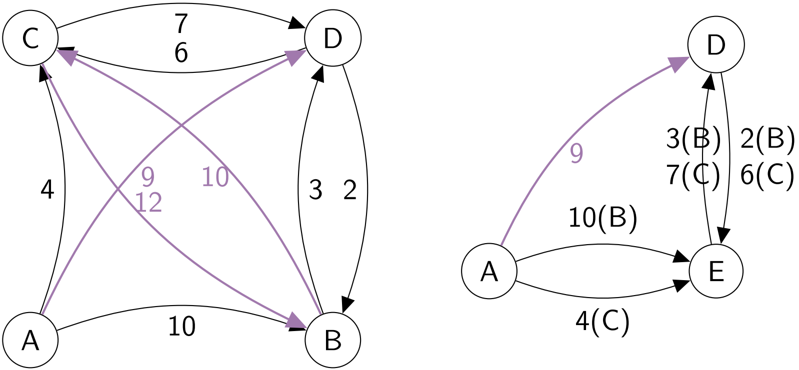
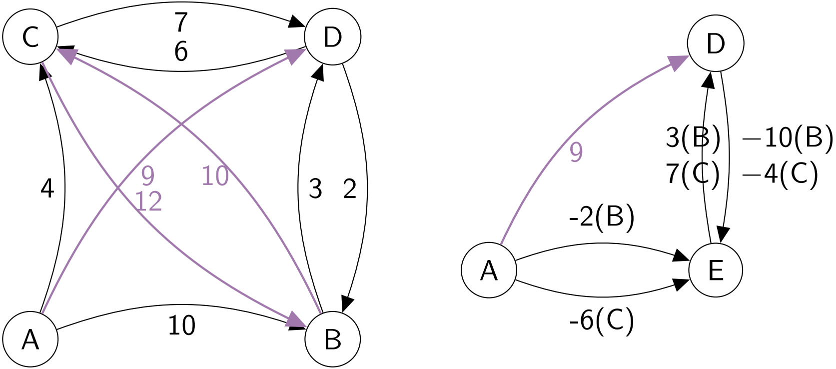
Step 3.1: Contract the loop (B-C) into one node, and create a new graph.



Running Chu-Liu/Edmonds' Algorithm with an Example

Step 3.2: Contract the loop (B-C) into one node, and create a new graph, with the adjusted weights.



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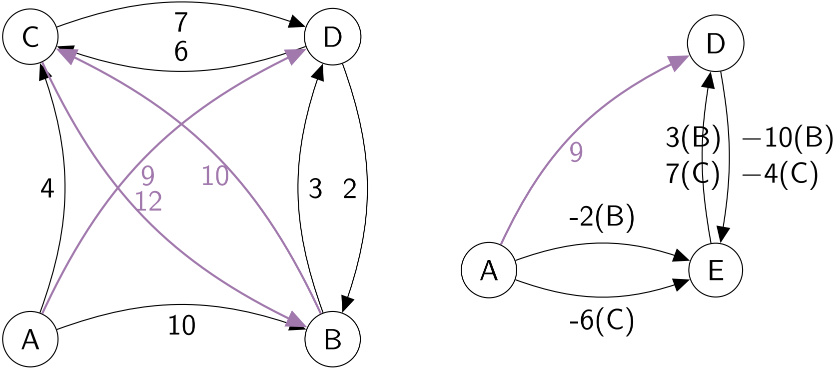

Repeat the process until we find the maximum spanning tree.

Complexity Analysis

- Time complexity: $O(EV)$ for dense graphs, can be improved to $O(E \log V)$ with optimized implementations.
- Space complexity: $O(E + V)$ to store the sparse graph, or $O(V^2)$ for dense graphs.
- Handles non-projective dependencies.

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When to use:

- Use Collins' (or Eisner's) when you can assume projectivity.
- Use Chu-Liu-Edmonds when non-projective structures are important.
- Many languages with free word order benefit from non-projective parsing.

Neural Dependency Parsers

Take (contextualized) word representations, and predict the scores between each pair of words.

Maximize the ground-truth arc scores in the training set.

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Now: `https://spacy.io/` offers high-quality off-the-shelf dependency parsers.

Next

Semantics: Compositionality, Semantic Role Labeling, Lambda Calculus